

AppellF1

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Notations

Traditional name

Appell hypergeometric function F_1

Traditional notation

$$F_1(a; b_1, b_2; c; z_1, z_2)$$

Mathematica StandardForm notationAppellF1[a, b_1, b_2, c, z_1, z_2]

Primary definition

07.36.02.0001.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(a)_{k+l} (b_1)_k (b_2)_l z_1^k z_2^l}{(c)_{k+l} k! l!} ; |z_1| < 1 \wedge |z_2| < 1$$

Specific values

Specialized values

Values at $(z_1, z_2) = (0, 0)$

07.36.03.0001.01

$$F_1(a; b_1, b_2; c; 0, 0) = 1$$

Values at $z_2 = 0$

07.36.03.0002.01

$$F_1(a; b_1, b_2; c; z, 0) = {}_2F_1(a, b_1; c; z)$$

Values at $z_2 = 1$

07.36.03.0003.01

$$F_1(a; b_1, b_2; c; z, 1) = {}_2F_1(a, b_2; c; 1) {}_2F_1(a, b_1; c - b_2; z)$$

Values at $z_2 = \pm z_1$

07.36.03.0004.01

$$F_1(a; b_1, b_2; c; z, z) = {}_2F_1(a, b_1 + b_2; c; z)$$

07.36.03.0005.01

$$F_1(a; b_1, b_1; c; z, -z) = {}_3F_2\left(\frac{a+1}{2}, \frac{a}{2}, b_1; \frac{c+1}{2}, \frac{c}{2}; z^2\right)$$

Values at fixed points

For fixed a, b_1, b_2, z_1, z_2

07.36.03.0006.01

$$F_1(a; b_1, b_2; b_1 + b_2; z_1, z_2) = (1 - z_2)^{-a} {}_2F_1\left(a, b_1; b_1 + b_2; \frac{z_1 - z_2}{1 - z_2}\right)$$

For fixed z_1, z_2

07.36.03.0011.01

$$F_1(a; b_1, b_2; a - n; z_1, z_2) - (1 - z_1)^{-b_1} (1 - z_2)^{-b_2} \sum_{k=0}^n \sum_{l=0}^{n-k} \frac{(-n)_{k+l} (b_1)_k (b_2)_l}{(a - n)_{k+l} k! l!} \left(\frac{z_1}{z_1 - 1}\right)^k \left(\frac{z_2}{z_2 - 1}\right)^l; n \in \mathbb{N}$$

07.36.03.0012.01

$$F_1\left(n + \frac{1}{2}; n - \frac{1}{2}, \frac{1}{2}; n + \frac{3}{2}; z_1, z_2\right) = \frac{(2n+1) z_2^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)_n} \frac{\partial^n E\left(\sin^{-1}(\sqrt{z_2}) \middle| \frac{z_1}{z_2}\right)}{\partial z_1^n}$$

07.36.03.0013.01

$$F_1\left(n + \frac{1}{2}; n + \frac{1}{2}, \frac{1}{2}; n + \frac{3}{2}; z_1, z_2\right) = \frac{2n+1}{\left(\frac{1}{2}\right)_n \sqrt{z_2}} \frac{\partial^n F\left(\sin^{-1}(\sqrt{z_2}) \middle| \frac{z_1}{z_2}\right)}{\partial z_1^n}; n \in \mathbb{N}$$

07.36.03.0014.01

$$F_1\left(n + \frac{1}{2}; n + 1, 1; n + \frac{3}{2}; z_1, z_2\right) =$$

$$\frac{2n+11}{2n!} \sum_{m=0}^n \frac{(-1)^{n-m} (m)_{2(n-m)}}{(n-m)! (2\sqrt{z_1})^{2n-m}} \left[(-1)^m \tanh^{-1}(\sqrt{z_1}) m! \left((\sqrt{z_1} - \sqrt{z_2})^{-m-1} + (\sqrt{z_1} + \sqrt{z_2})^{-m-1} \right) + \right.$$

$$\left. \sum_{j=0}^m \binom{m}{j} (-1)^{m-j} (m-j)! \left(\frac{1}{(\sqrt{z_1} + \sqrt{z_2})^{-j+m+1}} + \frac{1}{(\sqrt{z_1} - \sqrt{z_2})^{-j+m+1}} \right) \right]$$

$$\sum_{k=0}^{j-1} \frac{k! (-j+2k+2)_{2(j-k-1)} (1-z_1)^{-k-1}}{(j-k-1)! (2\sqrt{z_1})^{j-2k-1}} \Bigg] - (-1)^n (2n+1) \sqrt{z_2} \tanh^{-1}(\sqrt{z_2}) (z_1 - z_2)^{-n-1}; n \in \mathbb{N}$$

07.36.03.0007.01

$$F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{2}; \frac{3}{2}; z_1, z_2\right) = \frac{1}{\sqrt{z_1}} E\left(\sin^{-1}(\sqrt{z_1}) \middle| \frac{z_2}{z_1}\right)$$

07.36.03.0008.01

$$F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z_1, z_2\right) = \frac{1}{\sqrt{z_1}} F\left(\sin^{-1}(\sqrt{z_1}) \middle| \frac{z_2}{z_1}\right)$$

07.36.03.0009.01

$$F_1\left(\frac{3}{2}; \frac{1}{2}, \frac{1}{2}, \frac{3}{2}; z_1, z_2\right) = \frac{1}{\sqrt{z_1}} \operatorname{sn}^{-1}\left(\sqrt{z_1} \mid \frac{z_2}{z_1}\right)$$

07.36.03.0015.01

$$F_1\left(\frac{1}{2}; 1, 1; \frac{3}{2}; z_1, z_2\right) = \frac{\tanh^{-1}(\sqrt{z_1}) \sqrt{z_1} - \tanh^{-1}(\sqrt{z_2}) \sqrt{z_2}}{z_1 - z_2}$$

07.36.03.0016.01

$$F_1\left(\frac{3}{2}; \frac{1}{2}, 1; \frac{5}{2}; z_1, z_2\right) = \frac{3}{\sqrt{z_1} z_2} \left(\frac{1}{\sqrt{\frac{z_2 - z_1}{z_1}}} \tanh^{-1}\left(\frac{\sqrt{z_1}}{\sqrt{1 - z_1}} \sqrt{\frac{z_2 - z_1}{z_1}}\right) - \sin^{-1}(\sqrt{z_1}) \right)$$

07.36.03.0017.01

$$F_1\left(\frac{5}{2}; \frac{1}{2}, 1; \frac{7}{2}; z_1, z_2\right) = \frac{5}{4 \sqrt{z_1} z_2^2} \left(\frac{4}{\sqrt{\frac{z_2 - z_1}{z_1}}} \tanh^{-1}\left(\frac{\sqrt{z_1}}{\sqrt{1 - z_1}} \sqrt{\frac{z_2 - z_1}{z_1}}\right) - 2 \sin^{-1}(\sqrt{z_1}) \left(\frac{z_2}{z_1} + 2\right) + \frac{2 \sqrt{1 - z_1} z_2}{\sqrt{z_1}} \right)$$

07.36.03.0018.01

$$F_1\left(n + \frac{1}{2}; \frac{1}{2}, 1; n + \frac{3}{2}; z_1, z_2\right) = \frac{2n + 1}{\sqrt{z_1}} \left(\frac{z_2^{-n}}{\sqrt{\frac{z_2}{z_1} - 1}} \tanh^{-1}\left(\frac{\sqrt{z_1} \sqrt{\frac{z_2}{z_1} - 1}}{\sqrt{1 - z_1}}\right) - 2^{-n} z_1^{-n} \sum_{j=0}^{n-1} (-1)^j \binom{n}{j+1} \left(\frac{z_2 - 2z_1}{z_2}\right)^{j+1} \sum_{i=0}^{j+1} \binom{j+1}{i} z_1^i (z_2 - 2z_1)^{-i} \left(\sum_{q=0}^{\lfloor \frac{i-1}{2} \rfloor} \frac{1}{q!} \right. \right. \\ \left. \left. \left(\binom{i-1}{2q} \left(\frac{1}{2}\right)_q z_1^{-2q} \left(\frac{2z_1 - z_2}{z_1}\right)^{i-2q-1} z_2^{2q} \left(2 \sin^{-1}(\sqrt{z_1}) + \sqrt{1 - z_1} \sqrt{z_1} \sum_{p=1}^q \frac{(p-1)! (1 - 2z_1)^{2p-1}}{\left(\frac{1}{2}\right)_p} \right) \right) \right. \right. \\ \left. \left. + \sum_{q=0}^{\lfloor \frac{i-1}{2} \rfloor - 1} 2^{2q+1} \binom{i-1}{2q+1} q! (1 - z_1)^{q+\frac{1}{2}} z_1^{-q-\frac{1}{2}} \left(\frac{2z_1 - z_2}{z_1}\right)^{i-2q-2} z_2^{2q+1} \sum_{p=0}^q \frac{2^{-2p} (1 - 2z_1)^{2p} (1 - z_1)^{-p} z_1^{-p}}{p! \left(\frac{3}{2}\right)_{q-p}} \right) \right)$$

07.36.03.0019.01

$$F_1\left(\frac{3}{2}; 2, 1; \frac{5}{2}; z_1, z_2\right) = \frac{3}{2(1 - z_1) \sqrt{z_1} (z_1 - z_2)^2} \left(\sqrt{z_1} \left(-2 \tanh^{-1}(\sqrt{z_2}) \sqrt{z_2} (z_1 - 1) + z_1 - z_2 \right) + \tanh^{-1}(\sqrt{z_1}) (z_1 - 1) (z_1 + z_2) \right)$$

07.36.03.0020.01

$$F_1\left(\frac{5}{2}; 3, 1; \frac{7}{2}; z_1, z_2\right) = \frac{5}{8(z_1 - 1)^2 z_1^{3/2} (z_1 - z_2)^3} \left(\tanh^{-1}(\sqrt{z_1}) (3z_1^2 + 6z_2 z_1 - z_2^2) (z_1 - 1)^2 + \sqrt{z_1} (5z_1^3 - 3(2z_2 + 1)z_1^2 + z_2(z_2 + 2)z_1 - 8 \tanh^{-1}(\sqrt{z_2}) (z_1 - 1)^2 \sqrt{z_2} z_1 + z_2^2) \right)$$

07.36.03.0010.01

$$F_1\left(\frac{1}{2}; \frac{1}{2}, 1; 1; z_1, z_2\right) = \frac{2}{\pi} \Pi(z_2 | z_1)$$

07.36.03.0021.01

$$F_1\left(1; -\frac{1}{2}, 1; n + \frac{3}{2}; z_1, z_2\right) = (2n+1) \left((-1)^{n-1} \sqrt{\frac{z_2-z_1}{z_2-1}} \tanh^{-1}\left(\sqrt{\frac{z_2-z_1}{z_2-1}}\right) (1-z_2)^n z_2^{-n-1} + \frac{(-1)^n (1-z_1)^{n+1}}{n! (1-z_2)} \right. \\ \left. \sum_{m=0}^{n-1} \frac{(m+1) 2^{m-2n} z_1^{\frac{m+1}{2}-n} (2n-m-2)!}{(n-m-1)!} \left(\left(\sqrt{z_1} - \sqrt{\frac{z_2-z_1}{z_2-1}} \right)^{-m-2} + \left(\sqrt{\frac{z_2-z_1}{z_2-1}} + \sqrt{z_1} \right)^{-m-2} \right) \tanh^{-1}(\sqrt{z_1}) - \right. \\ \left. \sum_{j=0}^m \frac{(-1)^j}{j+1} \left(\left(\sqrt{z_1} - \sqrt{\frac{z_2-z_1}{z_2-1}} \right)^{j-m-1} + \left(\sqrt{\frac{z_2-z_1}{z_2-1}} + \sqrt{z_1} \right)^{j-m-1} \right) \sum_{k=0}^j \frac{2^{j-2k} (1-z_1)^{-j+k-1} z_1^{\frac{j}{2}-k} (j-k)!}{(j-2k)! k!} \right) \right)$$

General characteristics

Some abbreviations

07.36.04.0001.01

$$\mathcal{NT}(\{a_1, a_2\}) = \neg (-a_1 \in \mathbb{N} \vee -a_2 \in \mathbb{N})$$

Domain and analyticity

$F_1(a; b_1, b_2; c; z_1, z_2)$ is an analytical function of a, b_1, b_2, c, z_1, z_2 , which is defined in $\mathbb{C}^6$. For fixed a, b_1, c, z_1, z_2 , it is an entire function of b_2 . For fixed a, b_2, c, z_1, z_2 , it is an entire function of b_1 . For fixed b_1, b_2, c, z_1, z_2 , it is an entire function of a . For negative integer a , $F_1(a; b_1, b_2; c; z_1, z_2)$ degenerates to a polynomial in z_1, z_2 . For negative integer b_k , $F_1(a; b_1, b_2; c; z_1, z_2)$ degenerates to a polynomial in z_k of order $-b_k$, $1 \leq k \leq 2$.

07.36.04.0002.01

$$(a * b_1 * b_2 * c * z_1 * z_2) \longrightarrow F_1(a; b_1, b_2; c; z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.36.04.0003.01

$$F_1(\bar{a}; \bar{b}_1, \bar{b}_2; \bar{c}; \bar{z}_1, \bar{z}_2) = \overline{F_1(a; b_1, b_2; c; z_1, z_2)} /; z_1 \notin (1, \infty) \wedge z_2 \notin (1, \infty)$$

Permutation symmetry

07.36.04.0004.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = F_1(a; b_2, b_1; c; z_2, z_1)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z_k

For fixed a, b_k, c , in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \vee -b_k \in \mathbb{N})$), the function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have poles and essential singularities with respect to z_k , $1 \leq k \leq 2$.

07.36.04.0005.01

$$\text{Sing}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\} /; \mathcal{NT}(\{a, b_k\}) \wedge 1 \leq k \leq 2$$

For negative integer a or b_k and fixed $c, z_j, j \neq k$, the function $F_1(a; b_1, b_2; c; z_1, z_2)$ is a polynomial and has pole of order $-a$ or $-b_k$ at $z_k = \infty$, $1 \leq k \leq 2$.

07.36.04.0006.01

$$\begin{aligned} \text{Sing}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2)) &= \{\{\infty, -a\}\} /; \\ &(-a \in \mathbb{N}^+ \wedge a = a) \vee (-b_k \in \mathbb{N}^+ \wedge a = b_k) \vee (-a \in \mathbb{N}^+ \wedge -b_k \in \mathbb{N}^+ \wedge a = \min(-a, -b_k)) \wedge 1 \leq k \leq 2 \end{aligned}$$

With respect to c

For fixed a, b_1, b_2, z_1, z_2 , the function $F_1(a; b_1, b_2; c; z_1, z_2)$ has an infinite set of singular points with respect to c :

- a) the points $c = -k /; k \in \mathbb{N}$ are the simple poles with residues $\frac{(-1)^k}{k!} {}_2\tilde{F}_1(a, b; -k; z)$;
- b) $c = \infty$ is the point of convergence of poles, which is an essential singular point.

07.36.04.0007.01

$$\text{Sing}_c(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\{-k, 1\} /; k \in \mathbb{N}\}, \{\infty, \infty\}$$

07.36.04.0008.01

$$\text{res}_c(F_1(a; b_1, b_2; c; z_1, z_2))(-k) = \frac{(-1)^k}{k!} {}_2\tilde{F}_1(a, b; -k; z) /; k \in \mathbb{N}$$

With respect to b_k

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have poles and essential singularities with respect to b_k , $1 \leq k \leq 2$.

07.36.04.0009.01

$$\text{Sing}_{b_k}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\} /; k \in \{1, 2\}$$

With respect to a

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have poles and essential singularities with respect to a .

07.36.04.0010.01

$$\text{Sing}_a(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\}$$

Branch points**With respect to z_k**

For fixed a, b_k, c , in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \vee -b_k \in \mathbb{N})$), the function $F_1(a; b_1, b_2; c; z_1, z_2)$ has for fixed z_1 or fixed z_2 two singular branch points with respect to z_2 or z_1 : $z_k = 1, z_k = \infty, k = 1, 2$.

07.36.04.0011.01

$$\mathcal{BP}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{1, \infty\} /; \mathcal{NT}(\{a, b_k\}) \wedge k \in \{1, 2\}$$

07.36.04.0012.01

$$\mathcal{R}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2), 1) = \log /; c - a - b_k \in \mathbb{Z} \vee c - a - b_k \notin \mathbb{Q} \wedge \mathcal{NT}(\{a, b_k\}) \wedge k \in \{1, 2\}$$

07.36.04.0013.01

$$\mathcal{R}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2), 1) = s /; c - a - b_k = \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) = 1 \bigwedge \mathcal{NT}(\{a, b_k\}) \bigwedge k \in \{1, 2\}$$

07.36.04.0014.01

$$\mathcal{R}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2), \infty) = \log /; a - b_k \in \mathbb{Z} \vee \neg (a \in \mathbb{Q} \wedge b_k \in \mathbb{Q}) \wedge k \in \{1, 2\}$$

07.36.04.0015.01

$$\mathcal{R}_{z_k}(F_1(a; b_1, b_2; c; z_1, z_2), \infty) = \text{lcm}(s, u) /; a = \frac{r}{s} \bigwedge b_k = \frac{t}{u} \bigwedge \{r, s, t, u\} \in \mathbb{Z} \bigwedge s > 0 \bigwedge u > 0 \bigwedge \text{lcm}(s, u) \neq 1 \bigwedge \gcd(r, s) = 1 \bigwedge \gcd(t, u) = 1 \bigwedge \mathcal{NT}(\{a, b_k\}) \bigwedge k \in \{1, 2\}$$

With respect to c

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch points with respect to c .

07.36.04.0016.01

$$\mathcal{BP}_c(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\}$$

With respect to b_k

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch points with respect to b_k , $1 \leq k \leq 2$.

07.36.04.0017.01

$$\mathcal{BP}_{b_k}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\} /; k \in \{1, 2\}$$

With respect to a

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch points with respect to a .

07.36.04.0018.01

$$\mathcal{BP}_a(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\}$$

Branch cuts**With respect to z_1**

For fixed a, b_1, c , in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \vee -b_1 \in \mathbb{N})$), the function $F_1(a; b_1, b_2; c; z_1, z_2)$ is a single-valued function on the z_1 -plane cut along the interval $(1, \infty)$, where it is continuous from below.

07.36.04.0019.01

$$\mathcal{BC}_{z_1}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{(1, \infty), i\} /; \mathcal{NT}(\{a, b_1\})$$

07.36.04.0020.01

$$\lim_{\epsilon \rightarrow +0} F_1(a; b_1, b_2; c; x_1 - i\epsilon, z_2) = F_1(a; b_1, b_2; c; x_1, z_2) /; x_1 > 1$$

07.36.04.0021.01

$$\lim_{\epsilon \rightarrow +0} F_1(a; b_1, b_2; c; x_1 + i\epsilon, z_2) =$$

$$e^{2i\pi(a-c+b_1)} F_1(a; b_1, b_2; c; x_1, z_2) + \frac{2i e^{i\pi(a-c+b_1)} \pi \Gamma(c)}{\Gamma(c-a) \Gamma(a-c+b_1+1) \Gamma(c-b_1)} F_{0 \times 1 \times 1}^{1 \times 1 \times 1} \left(\begin{matrix} a; b_1, b_2; \\ a-c+b_1+1; c-b_1; \end{matrix} \middle| 1-x_1, z_2 \right) /; x_1 > 1$$

With respect to z_2

For fixed a, b_2, c , in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \vee -b_2 \in \mathbb{N})$), the function $F_1(a; b_1, b_2; c; z_1, z_2)$ is a single-valued function on the z_2 -plane cut along the interval $(1, \infty)$, where it is continuous from below.

07.36.04.0022.01

$$\mathcal{BC}_{z_2}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{(1, \infty), i\} /; \mathcal{NT}(\{a, b_2\})$$

07.36.04.0023.01

$$\lim_{\epsilon \rightarrow +0} F_1(a; b_1, b_2; c; z_1, x_2 - i\epsilon) = F_1(a; b_1, b_2; c; z_1, x_2) /; x_2 > 1$$

07.36.04.0024.01

$$\lim_{\epsilon \rightarrow +0} F_1(a; b_1, b_2; c; z_1, x_2 + i\epsilon) =$$

$$e^{2i\pi(a-c+b_2)} F_1(a; b_1, b_2; c; z_1, x_2) + \frac{2i e^{i\pi(a-c+b_2)} \pi \Gamma(c)}{\Gamma(c-a) \Gamma(a-c+b_2+1) \Gamma(c-b_2)} F_{0 \times 1 \times 1}^{1 \times 1 \times 1} \left(\begin{matrix} a; b_1; b_2; \\ c-b_2; 1+a-c+b_2; \end{matrix} \middle| z_1, 1-x_2 \right) /; x_2 > 1$$

With respect to c

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch cuts with respect to c .

07.36.04.0025.01

$$\mathcal{BC}_c(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\}$$

With respect to b_k

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch cuts with respect to b_k , $1 \leq k \leq 2$.

07.36.04.0026.01

$$\mathcal{BC}_{b_k}(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\} /; k \in \{1, 2\}$$

With respect to a

The function $F_1(a; b_1, b_2; c; z_1, z_2)$ does not have branch cuts with respect to a .

07.36.04.0027.01

$$\mathcal{BC}_a(F_1(a; b_1, b_2; c; z_1, z_2)) = \{\}$$

Series representations

Generalized power series

Expansions at $(z_1, z_2) = (0, 0)$

For the function itself

07.36.06.0001.01

$$\begin{aligned} F_1(a; b_1, b_2; c; z_1, z_2) \propto & 1 + \frac{a b_1 z_1}{c} + \frac{a(1+a) b_1(1+b_1) z_1^2}{2c(1+c)} + \frac{a b_2 z_2}{c} + \frac{a(1+a) b_1 b_2 z_1 z_2}{c(1+c)} + \\ & \frac{a(1+a)(2+a) b_1(1+b_1) b_2 z_1^2 z_2}{2c(1+c)(2+c)} + \frac{a(1+a) b_2(1+b_2) z_2^2}{2c(1+c)} + \frac{a(1+a)(2+a) b_1 b_2(1+b_2) z_1 z_2^2}{2c(1+c)(2+c)} + \\ & \frac{a(1+a)(2+a)(3+a) b_1(1+b_1) b_2(1+b_2) z_1^2 z_2^2}{4c(1+c)(2+c)(3+c)} + \dots /; (z_1 \rightarrow 0) \wedge (z_2 \rightarrow 0) \end{aligned}$$

07.36.06.0002.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(a)_{k+l} (b_1)_k (b_2)_l z_1^k z_2^l}{(c)_{k+l} k! l!} ; |z_1| < 1 \wedge |z_2| < 1$$

Expansions at $z_1 = 0$ **For the function itself**

07.36.06.0003.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_2; c; z_2) + \frac{a b_1 {}_2F_1(a+1, b_2; c+1; z_2)}{c} z_1 + \frac{a(a+1) b_1 (b_1+1) {}_2F_1(a+2, b_2; c+2; z_2)}{2c(c+1)} z_1^2 + \dots ; (z_1 \rightarrow 0)$$

07.36.06.0022.01

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_2; c; z_2) + \frac{a b_1 {}_2F_1(a+1, b_2; c+1; z_2)}{c} z_1 + \frac{a(a+1) b_1 (b_1+1) {}_2F_1(a+2, b_2; c+2; z_2)}{2c(c+1)} z_1^2 + O(z_1^3)$$

07.36.06.0004.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \frac{(a)_k (b_1)_k}{(c)_k k!} {}_2F_1(a+k, b_2; c+k; z_2) z_1^k ; |z_1| < 1$$

07.36.06.0005.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_2; c; z_2) (1 + O(z_1))$$

07.36.06.0023.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = F_{\infty}(z_1, a, b_1, b_2, c, z_2) ;$$

$$\left(\left(F_n(z_1, a, b_1, b_2, c, z_2) = \sum_{k=0}^n \frac{(a)_k (b_1)_k z_1^k}{(c)_k k!} {}_2F_1(a+k, b_2; c+k; z_2) = F_1(a; b_1, b_2; c; z_1, z_2) - \right. \right.$$

$$\left. \left. \frac{(a)_{n+1} (b_1)_{n+1} z_1^{n+1}}{(c)_{n+1} (n+1)!} F_{1 \times 2 \times 1}^{1 \times 2 \times 0} \left(\begin{matrix} a+n+1; 1, b_1+n+1; b_2; \\ c+n+1; n+2; ; \end{matrix} ; z_1, z_2 \right) \right) \bigg| \bigwedge n \in \mathbb{N} \right)$$

Summed form of the truncated series expansion.

Expansions at $z_2 = 0$ **For the function itself**

07.36.06.0006.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_1; c; z_2) + \frac{a b_2 {}_2F_1(a+1, b_1; c+1; z_1)}{c} z_2 + \frac{a(a+1) b_2 (b_2+1) {}_2F_1(a+2, b_1; c+2; z_1)}{2c(c+1)} z_2^2 + \dots ; (z_2 \rightarrow 0)$$

07.36.06.0024.01

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_1; c; z_2) + \frac{a b_2 {}_2F_1(a+1, b_1; c+1; z_1)}{c} z_2 + \frac{a(a+1) b_2 (b_2+1) {}_2F_1(a+2, b_1; c+2; z_1)}{2c(c+1)} z_2^2 + O(z_2^3)$$

07.36.06.0007.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \frac{(a)_k (b_2)_k}{(c)_k k!} {}_2F_1(a+k, b_1; c+k; z_1) z_2^k /; |z_2| < 1$$

07.36.06.0008.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto {}_2F_1(a, b_1; c; z_1) (1 + O(z_2))$$

07.36.06.0025.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = F_{\infty}(z_2, a, b_1, b_2, c, z_1) /;$$

$$\left(\left(F_n(z_2, a, b_1, b_2, c, z_1) = \sum_{k=0}^n \frac{(a)_k (b_2)_k z_2^k}{(c)_k k!} {}_2F_1(a+k, b_1; c+k; z_1) = F_1(a; b_1, b_2; c; z_1, z_2) - \frac{(a)_{n+1} (b_2)_{n+1} z_2^{n+1}}{(c)_{n+1} (n+1)!} F_{1 \times 2 \times 1}^{1 \times 2 \times 1} \left(\begin{matrix} a+n+1; 1, b_1; b_2+n+1; \\ c+n+1; n+2; \end{matrix} z_1, z_2 \right) \right) \bigg| \bigwedge n \in \mathbb{N} \right)$$

Summed form of the truncated series expansion.

Expansions at $z_1 = 1$ **For the function itself**

07.36.06.0009.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(a-c+b_1)}{\Gamma(a) \Gamma(b_1)} (1-z_1)^{c-a-b_1} \sum_{k=0}^{\infty} \frac{(b_2)_k}{k!} {}_2F_1(c-a, c+k-b_1; c-a-b_1+1; 1-z_1) z_2^k + \frac{\Gamma(c) \Gamma(c-a-b_1)}{\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k (b_2)_k}{k! \Gamma(c+k-b_1)} {}_2F_1(a+k, b_1; a-c+b_1+1; 1-z_1) z_2^k$$

07.36.06.0010.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(a+b_1-c)}{\Gamma(a) \Gamma(b_1)} (1-z_1)^{c-a-b_1} F_{0 \times 1 \times 1}^{11 \times 1} \left(\begin{matrix} c-b_1; c-a; b_2; \\ c-a-b_1+1; c-b_1; \end{matrix} 1-z_1, z_2 \right) + \frac{\Gamma(c) \Gamma(c-a-b_1)}{\Gamma(c-a) \Gamma(c-b_1)} F_{0 \times 1 \times 1}^{11 \times 1} \left(\begin{matrix} a; b_1; b_2; \\ a+b_1-c+1; c-b_1; \end{matrix} 1-z_1, z_2 \right)$$

07.36.06.0011.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto \frac{\Gamma(c) \Gamma(a+b_1-c) (1-z_2)^{-b_2}}{\Gamma(a) \Gamma(b_1)} (1-z_1)^{c-a-b_1} (1 + O(z_1-1)) + \frac{\Gamma(c) \Gamma(c-a-b_1)}{\Gamma(c-a) \Gamma(c-b_1)} {}_2F_1(a, b_2; c-b_1; z_2) (1 + O(z_1-1))$$

Expansions at $z_2 = 1$ **For the function itself**

07.36.06.0012.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(a-c+b_2)}{\Gamma(a) \Gamma(b_2)} (1-z_2)^{c-a-b_2} \sum_{k=0}^{\infty} \frac{(b_1)_k}{k!} {}_2F_1(c-a, c+k-b_2; c-a-b_2+1; 1-z_2) z_1^k + \frac{\Gamma(c) \Gamma(c-a-b_2)}{\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k (b_1)_k}{k! \Gamma(c+k-b_2)} {}_2F_1(a+k, b_2; a-c+b_2+1; 1-z_2) z_1^k$$

07.36.06.0013.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(a + b_2 - c)}{\Gamma(a) \Gamma(b_2)} (1 - z_2)^{c-a-b_2} F_{0 \times 1 \times 1}^{11 \times 1} \left(\begin{matrix} c - b_2; b_1; c - a; \\ c - b_2; c - a - b_2 + 1; \end{matrix} ; z_1, 1 - z_2 \right) +$$

$$\frac{\Gamma(c) \Gamma(c - a - b_2)}{\Gamma(c - a) \Gamma(c - b_2)} F_{0 \times 1 \times 1}^{11 \times 1} \left(\begin{matrix} a; b_1; b_2; \\ c - b_2; a + b_2 - c + 1; \end{matrix} ; z_1, 1 - z_2 \right)$$

07.36.06.0014.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto$$

$$\frac{\Gamma(c) \Gamma(a + b_2 - c) (1 - z_1)^{-b_1}}{\Gamma(a) \Gamma(b_2)} (1 - z_2)^{c-a-b_2} (1 + O(z_2 - 1)) + \frac{\Gamma(c) \Gamma(c - a - b_2)}{\Gamma(c - a) \Gamma(c - b_2)} {}_2F_1(a, b_1; c - b_2; z_1) (1 + O(z_2 - 1))$$

Expansions at $z_1 = \infty$ **For the function itself**

07.36.06.0015.02

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(b_1 - a)}{\Gamma(c - a) \Gamma(b_1)} (-z_1)^{-a} F_1 \left(a; a - c + 1, b_2; a - b_1 + 1; \frac{1}{z_1}, \frac{z_2}{z_1} \right) +$$

$$\frac{\Gamma(c)}{\Gamma(a)} (-z_1)^{-b_1} \sum_{k=0}^{\infty} \frac{\Gamma(a + k - b_1) (b_2)_k}{k! \Gamma(c + k - b_1 + k)} {}_2F_1 \left(b_1, b_1 - c - k + 1; b_1 - a - k + 1; \frac{1}{z_1} \right) z_2^k /; a - b_1 \notin \mathbb{Z}$$

07.36.06.0016.02

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(b_1 - a)}{\Gamma(c - a) \Gamma(b_1)} (-z_1)^{-a} F_1 \left(a; b_2, a - c + 1; a - b_1 + 1; \frac{z_2}{z_1}, \frac{1}{z_1} \right) +$$

$$\frac{\Gamma(c) \Gamma(a - b_1)}{\Gamma(a) \Gamma(c - b_1)} (-z_1)^{-b_1} \left(F_{1 \times 1 \times 1}^{1 \times 2 \times 2} \left(\begin{matrix} b_1; b_2, a - b_1; b_1 - c + 1, 1; \\ 1; c - b_1; b_1 - a + 1; \end{matrix} ; \frac{z_2}{z_1}, \frac{1}{z_1} \right) + \right.$$

$$\left. \frac{(a - b_1)^2 b_2 z_2}{(c - b_1)^2} F_{2 \times 0 \times 1}^{2 \times 1 \times 2} \left(\begin{matrix} a - b_1 + 1, b_2 + 1; b_1; a - b_1 + 1, 1; \\ c - b_1 + 1, 2;; c - b_1 + 1; \end{matrix} ; \frac{z_2}{z_1}, z_2 \right) \right) /; a - b_1 \notin \mathbb{Z}$$

07.36.06.0017.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto \frac{\Gamma(c) \Gamma(b_1 - a)}{\Gamma(c - a) \Gamma(b_1)} (-z_1)^{-a} \left(1 + O\left(\frac{1}{z_1}\right) \right) + \frac{\Gamma(c) \Gamma(a - b_1)}{\Gamma(c - b_1) \Gamma(a)} (-z_1)^{-b_1} \left(1 + O\left(\frac{1}{z_1}\right) \right) /; a \neq b_1$$

Expansions at $z_2 = \infty$ **For the function itself**

07.36.06.0018.02

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(b_2 - a)}{\Gamma(c - a) \Gamma(b_2)} (-z_2)^{-a} F_1 \left(a; a - c + 1, b_1; a - b_2 + 1; \frac{1}{z_2}, \frac{z_1}{z_2} \right) +$$

$$\frac{\Gamma(c)}{\Gamma(a)} (-z_2)^{-b_2} \sum_{k=0}^{\infty} \frac{\Gamma(a + k - b_2) (b_1)_k}{k! \Gamma(c + k - b_2 + k)} {}_2F_1 \left(b_2, b_2 - c - k + 1; b_2 - a - k + 1; \frac{1}{z_2} \right) z_1^k /; a - b_2 \notin \mathbb{Z}$$

07.36.06.0019.02

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c) \Gamma(b_2 - a)}{\Gamma(c - a) \Gamma(b_2)} (-z_2)^{-a} F_1\left(a; b_1, a - c + 1; a - b_2 + 1; \frac{z_1}{z_2}, \frac{1}{z_2}\right) +$$

$$\frac{\Gamma(c) \Gamma(a - b_2)}{\Gamma(a) \Gamma(c - b_2)} (-z_2)^{-b_2} \left(F_{1 \times 1 \times 1}^{1 \times 2 \times 2}\left(b_2; b_1, a - b_2; b_2 - c + 1, 1; \frac{z_1}{z_2}, \frac{1}{z_2}\right) + \right.$$

$$\left. \frac{(a - b_2)^2 b_1 z_1}{(c - b_2)^2} F_{2 \times 0 \times 1}^{2 \times 1 \times 2}\left(a - b_2 + 1, b_1 + 1; b_2; a - b_2 + 1, 1; \frac{z_1}{z_2}, z_1\right) \right) /; a - b_2 \notin \mathbb{Z}$$

07.36.06.0020.02

$$F_1(a; b_1, b_2; c; z_1, z_2) \propto \frac{\Gamma(c) \Gamma(b_2 - a)}{\Gamma(c - a) \Gamma(b_2)} (-z_2)^{-a} \left(1 + O\left(\frac{1}{z_2}\right)\right) + \frac{\Gamma(c) \Gamma(a - b_2)}{\Gamma(c - b_2) \Gamma(a)} (-z_2)^{-b_2} \left(1 + O\left(\frac{1}{z_2}\right)\right) /; a \neq b_2$$

Residue representations

07.36.06.0021.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{\Gamma(a) \Gamma(b_1) \Gamma(b_2)} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \operatorname{res}_{s,t} \left(\frac{\Gamma(a - s - t) \Gamma(b_1 - s) \Gamma(b_2 - t) (-z_1)^{-s} (-z_2)^{-t}}{\Gamma(c - s - t)} \Gamma(s) \Gamma(t) \right) (-j, -k)$$

Integral representations

On the real axis

Of the direct function

07.36.07.0001.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{\Gamma(a) \Gamma(c - a)} \int_0^1 t^{a-1} (1 - t)^{c-a-1} (1 - t z_1)^{-b_1} (1 - t z_2)^{-b_2} dt /; \operatorname{Re}(a) > 0 \wedge \operatorname{Re}(c - a) > 0$$

Contour integral representations

07.36.07.0002.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{(2\pi i)^2 \Gamma(a) \Gamma(b_1) \Gamma(b_2)} \int_{\mathcal{L}^*} \int_{\mathcal{L}} \frac{\Gamma(a - s - t) \Gamma(s) \Gamma(b_1 - s) \Gamma(t) \Gamma(b_2 - t)}{\Gamma(c - s - t)} (-z_1)^{-s} (-z_2)^{-t} ds dt;$$

$$|\arg(-z_1)| < \pi \wedge |\arg(-z_2)| < \pi$$

Multiple integral representations

07.36.07.0003.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{\Gamma(c - b_1 - b_2) \Gamma(b_1) \Gamma(b_2)} \int_0^1 \int_0^{1-x} z_1^{b_1-1} z_2^{b_2-1} (-x - y + 1)^{c-b_1-b_2-1} (1 - x z_1 - y z_2)^{-a} dy dx /;$$

$$\operatorname{Re}(b_1) > 0 \wedge \operatorname{Re}(b_2) > 0 \wedge \operatorname{Re}(c - b_1 - b_2) > 0$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.36.13.0001.01

$$(1 - z_1) z_1 \frac{\partial^2 w(z_1, z_2)}{\partial z_1^2} + (c - (a + b_1 + 1) z_1) \frac{\partial w(z_1, z_2)}{\partial z_1} - b_1 z_2 \frac{\partial w(z_1, z_2)}{\partial z_2} + (1 - z_1) z_2 \frac{\partial^2 w(z_1, z_2)}{\partial z_1 \partial z_2} - a b_1 w(z_1, z_2) = 0 \wedge$$

$$(1 - z_2) z_2 \frac{\partial^2 w(z_1, z_2)}{\partial z_2^2} + (c - (a + b_2 + 1) z_2) \frac{\partial w(z_1, z_2)}{\partial z_2} - b_2 z_1 \frac{\partial w(z_1, z_2)}{\partial z_1} + (1 - z_2) z_1 \frac{\partial^2 w(z_1, z_2)}{\partial z_1 \partial z_2} - a b_2 w(z_1, z_2) = 0 /;$$

$$w(z_1, z_2) = F_1(a; b_1, b_2; c; z_1, z_2)$$

07.36.13.0002.01

$$(z_1 - 1) z_1 (z_1 - z_2) w^{(3,0)}(z_1, z_2) +$$

$$((b_1 + b_2 + 1)(z_1 - 1) z_1 + (a - c + b_1 + 2)(z_1 - z_2) z_1 + (c - b_2 + 1)(z_1 - 1)(z_1 - z_2)) w^{(2,0)}(z_1, z_2) + (b_1 + 1)$$

$$((2a + b_1 + 2) z_1 - (a - b_2 + 1) z_2 - c) w^{(1,0)}(z_1, z_2) + a b_1 (b_1 + 1) w(z_1, z_2) = 0 /; w(z_1, z_2) = F_1(a; b_1, b_2; c; z_1, z_2)$$

07.36.13.0003.01

$$(z_2 - z_1) (z_2 - 1) z_2 w^{(0,3)}(z_1, z_2) +$$

$$((c - b_1 + 1)(z_2 - z_1) (z_2 - 1) + (b_1 + b_2 + 1) z_2 (z_2 - 1) + (a - c + b_2 + 2)(z_2 - z_1) z_2) w^{(0,2)}(z_1, z_2) + (b_2 + 1)$$

$$(-c - (a - b_1 + 1) z_1 + (2a + b_2 + 2) z_2) w^{(0,1)}(z_1, z_2) + a b_2 (b_2 + 1) w(z_1, z_2) = 0 /; w(z_1, z_2) = F_1(a; b_1, b_2; c; z_1, z_2)$$

Identities

Functional identities

Major general cases

07.36.17.0001.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = (1 - z_1)^{-b_1} (1 - z_2)^{-b_2} F_1\left(c - a; b_1, b_2; c; \frac{z_1}{z_1 - 1}, \frac{z_2}{z_2 - 1}\right) /; z_1 \notin (1, \infty) \wedge z_2 \notin (1, \infty)$$

07.36.17.0002.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = (1 - z_1)^{-a} F_1\left(a; c - b_1 - b_2, b_2; c; \frac{z_1}{z_1 - 1}, \frac{z_1 - z_2}{z_1 - 1}\right) /; z_1 \notin (1, \infty) \wedge z_2 \notin (1, \infty)$$

07.36.17.0003.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = (1 - z_2)^{-a} F_1\left(a; b_1, c - b_1 - b_2; c; \frac{z_2 - z_1}{z_2 - 1}, \frac{z_2}{z_2 - 1}\right) /; z_1 \notin (1, \infty) \wedge z_2 \notin (1, \infty)$$

07.36.17.0004.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = (1 - z_1)^{c-a-b_1} (1 - z_2)^{-b_2} F_1\left(c - a; c - b_1 - b_2, b_2; c; z_1, \frac{z_2 - z_1}{z_2 - 1}\right)$$

07.36.17.0005.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = (1 - z_1)^{-b_1} (1 - z_2)^{c-a-b_2} F_1\left(c - a; b_1, c - b_1 - b_2; c; \frac{z_1 - z_2}{z_1 - 1}, z_2\right)$$

Differentiation

Low-order differentiation

With respect to z_1

07.36.20.0001.01

$$\frac{\partial F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_1} = \frac{a b_1}{c} F_1(a + 1; b_1 + 1, b_2; c + 1; z_1, z_2)$$

07.36.20.0002.01

$$\frac{\partial^2 F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_1^2} = \frac{a(a+1)b_1(b_1+1)}{c(c+1)} F_1(a+2; b_1+2, b_2; c+2; z_1, z_2)$$

With respect to z_2

07.36.20.0003.01

$$\frac{\partial F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_2} = \frac{a b_2}{c} F_1(a+1; b_1, b_2+1; c+1; z_1, z_2)$$

07.36.20.0004.01

$$\frac{\partial^2 F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_2^2} = \frac{a(a+1)b_2(b_2+1)}{c(c+1)} F_1(a+2; b_1, b_2+2; c+2; z_1, z_2)$$

Symbolic differentiation**With respect to z_1**

07.36.20.0005.02

$$\frac{\partial^n F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_1^n} = \frac{(a)_n (b_1)_n}{(c)_n} F_1(a+n; n+b_1, b_2; c+n; z_1, z_2) /; n \in \mathbb{N}$$

07.36.20.0009.01

$$\frac{\partial^n \left(z_1^{n+b_1-1} F_1(a; b_1, b_2; c; z_1, z_2) \right)}{\partial z_1^n} = (b_1)_n z_1^{b_1-1} F_1(a; n+b_1, b_2; c; z_1, z_2) /; n \in \mathbb{N}$$

With respect to z_2

07.36.20.0006.02

$$\frac{\partial^n F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_2^n} = \frac{(a)_n (b_2)_n}{(c)_n} F_1(a+n; b_1, b_2+n; c+n; z_1, z_2) /; n \in \mathbb{N}$$

07.36.20.0010.01

$$\frac{\partial^n \left(z_2^{n+b_2-1} F_1(a; b_1, b_2; c; z_1, z_2) \right)}{\partial z_2^n} = (b_2)_n z_2^{b_2-1} F_1(a; b_1, n+b_2; c; z_1, z_2) /; n \in \mathbb{N}$$

Fractional integro-differentiation**With respect to z_1**

07.36.20.0007.01

$$\frac{\partial^\alpha F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_1^\alpha} = z_1^{-\alpha} \Gamma(c) \tilde{F}_{1 \times 1 \times 0}^{1 \times 2 \times 1} \left(\begin{matrix} a; b_1, 1; b_2; \\ c; 1-\alpha; \end{matrix} ; z_1, z_2 \right)$$

With respect to z_2

07.36.20.0008.01

$$\frac{\partial^\alpha F_1(a; b_1, b_2; c; z_1, z_2)}{\partial z_2^\alpha} = z_2^{-\alpha} \Gamma(c) \tilde{F}_{1 \times 0 \times 1}^{1 \times 1 \times 2} \left(\begin{matrix} a; b_1; b_2, 1; \\ c; 1-\alpha; \end{matrix} ; z_1, z_2 \right)$$

Integration

Indefinite integration

Involving only one direct function with respect to z_1

07.36.21.0001.01

$$\int F_1(a; b_1, b_2; c; a z_1, z_2) dz_1 = \frac{c-1}{a(a-1)(b_1-1)} F_1(a-1; b_1-1, b_2; c-1; a z_1, z_2)$$

07.36.21.0002.01

$$\int F_1(a; b_1, b_2; c; z_1, z_2) dz_1 = \frac{c-1}{(a-1)(b_1-1)} F_1(a-1; b_1-1, b_2; c-1; z_1, z_2)$$

Involving only one direct function with respect to z_2

07.36.21.0003.01

$$\int F_1(a; b_1, b_2; c; z_1, a z_2) dz_2 = \frac{c-1}{a(a-1)(b_1-1)} F_1(a-1; b_1-1, b_2; c-1; z_1, a z_2)$$

07.36.21.0004.01

$$\int F_1(a; b_1, b_2; c; z_1, z_2) dz_2 = \frac{c-1}{(a-1)(b_2-1)} F_1(a-1; b_1, b_2-1; c-1; z_1, z_2)$$

Representations through more general functions

Through hypergeometric functions of two variables

07.36.26.0001.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = F_{1 \times 0 \times 0}^{1 \times 1 \times 1} \left(\begin{matrix} a; b_1; b_2; \\ c; \end{matrix} \middle| z_1, z_2 \right)$$

07.36.26.0002.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \Gamma(c) \tilde{F}_{1 \times 0 \times 0}^{1 \times 1 \times 1} \left(\begin{matrix} a; b_1; b_2; \\ c; \end{matrix} \middle| z_1, z_2 \right)$$

Through Meijer G

Classical cases for the direct function itself

07.36.26.0003.01

$$F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b_1)\Gamma(b_2)} G_{1,1,1,1,1,1}^{0,1,1,1,1,1} \left(\begin{matrix} 1-a & 1-b_1 & 1-b_2 \\ 1-c & 0 & 0 \end{matrix} \middle| -z_1, -z_2 \right)$$

History

- L. Pochhammer (1870)
- P. E. Appell (1880)
- J. Horn (1889)
- P.E. Appell and J. Kampé de Fériet (1926)

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